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STRATIFIED TWO-PHASE FLOW IN CIRCULAR PIPES

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Abstract—The paper discusses the stratified flow of liquid and gas in a circular pipe under the condition that the two components do not react with each other. If mass flow occurs across the interphase the influence of the entrainment of liquid droplets into the gas or the redeposition of them on the walls or on the interphase is accounted for by a lump parameter expressing the integrated effect of this phenomenon. The inclusion of this feature of the flow is shown to have a dramatic effect on the normal depth of the liquid phase under given conditions. Explicit analytical equations for the pressure drop and conditions determining the shape of the interphase including possible 'hydraulic jumps' are given. A special feature of the approach is the way in which the variation of the pressure in the liquid phase due to gravity is accounted for. The results in the paper may be used as a basis for experimental evaluation of the integrated shear stress at the interphase.

NOMENCLATURE

f	friction coefficient
f_i	friction coefficient at the interphase
F_G	part of the cross section filled by gas
F_L	part of the cross section filled by liquid
h	liquid depth at the vertical diameter
$h_m^{(G)}$	head loss for the gas flow
$h_m^{(L)}$	head loss for the liquid flow
p_G	pressure in the gas
p_L	pressure in the liquid
P_G	wetted periphery for the gas
P_L	wetted periphery for the liquid
Q_G	volume discharge of gas
Q_L	volume discharge of liquid
r	radius of the pipe
R_G	hydraulic radius for the gas
R_L	hydraulic radius for the liquid
s_i	length of interphase
v_G	mean velocity of gas
v_L	mean velocity of liquid
y	depth from liquid–gas interphase to streamline
z	height to streamline
z_0	height to bottom of pipe.

Greek symbols

α	inclination angle of the pipe
γ_G	specific weight of gas
γ_L	specific weight of liquid

Note: All lengths are made dimensionless through division by the radius r of the pipe. All dimensionless quantities are denoted by the superscript $+$.

INTRODUCTION

WHEN gas and liquid are simultaneously flowing in a pipe, a series of different flow patterns may occur. In horizontal or almost horizontal pipes the pattern described as stratified flow is schematically sketched in Fig. 1 and this type of flow is very often considered to be similar to that in an open channel. This presentation utilizes this analogy and applies the considerations usually advanced in channel flow. The objective is to first determine the normal depth h for given initial conditions, second to distinguish between subcritical and supercritical liquid flow whereby different behaviour of the interphase is observed, and third to express the pressure drop along the pipe in a single analytic expression. The relevance to the profession is that the whole treatment is non-dimensional and thus applicable to any case irrespective of pipe size.

The present study presents the results as depending on four parameters:

P_1 = ratio between the volumetric flow of gas Q_G and the volumetric flow of liquid Q_L ($P_1 = Q_G/Q_L$).
P_2 = ratio between the specific weights of gas γ_G and that of liquid γ_L ($P_2 = \gamma_G/\gamma_L$).
P_4 = ratio between the friction coefficient f_i of the interphase and that of the walls f ($P_4 = f_i/f$).
$h_0^+ = fQ_L^2/4gr^5 \sin\alpha$ where $\sin\alpha$ is the slope of the pipe and h_0^+ is a non-dimensional measure of the total flow.

The dependence of the normal depth h_n on these parameters is given. It is also shown how the depth h will change if for some reason it is brought away from this depth. This change is different according to whether or not the flow is subcritical or supercritical as demonstrated in Fig. 5 and this also explains the reason for a certain type of waviness of the interphase which may occur.

The study also presents the pressure drop in the pipe as a function of the parameters mentioned. The most

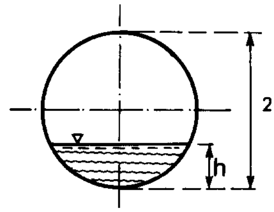

FIG. 1. Stratified flow in a pipe carrying gas and liquid.

important feature of the study is its non-dimensional form which makes the results applicable to all pipe sizes, a situation which is not always achieved in two-phase flow investigations.

THE BASIC EQUATIONS

The basic equations of stratified flow have been treated by Taitel and Duckler [1] and in keeping with their approach a one-dimensional (1-D) pipeline approach will be adopted. The flow in channels may be of three different types: (1) the uniform flow, where the flow is identical in all cross sections along the pipe, (2) the steady but accelerated flow, where the flow in a cross section is independent of time but where it changes from one cross section to another, (3) the non-steady flow. The second of these types of flow may be rather unstable and difficult to maintain experimentally but will be used here to identify the important parameters of the flow.

It will be assumed that the flow of the liquid can be adequately described by the energy equation for steady flow

$$\frac{v_L^2}{2g}+y+\frac{p_G}{\gamma_L}+z+h_m^{(L)}=\text{const.} \qquad (1)$$

These terms are shown in Fig. 2 and it is observed that $y+z=h+z_0$ where y is the distance from the liquid surface to the streamline, z is the height of the streamline, h is the depth of the liquid, and z_0 is the height of the bottom of the pipe. Equation (6) may thus be rearranged and differentiated rendering

$$\frac{d}{dx}\left[\frac{v_L^2}{2g}\right]+\frac{dh}{dx}-\sin\alpha+\frac{1}{\gamma_L}\frac{dp_G}{dx}+\frac{dh_m^{(L)}}{dx}=0 \qquad (2)$$

where

$$\frac{dz_0}{dx}=\sin\alpha.$$

It will furthermore be assumed that the flow of the gas may also be adequately described by the energy equation. However, the influence of gravity on the gas phase will be neglected, thus

$$\frac{v_G}{2g}+\frac{p_G}{\gamma_G}+h_m^{(G)}=\text{const.} \qquad (3)$$

Differentiation of this equation leaves

$$\frac{d}{dx}\left[\frac{v_G}{2g}\right]+\frac{1}{\gamma_G}\frac{dp_G}{dx}+\frac{dh_m^{(G)}}{dx}=0. \qquad (4)$$

Elimination of dp_G/dx between equations (2) and (4) will give the basic equation

$$\frac{d}{dx}\left[\frac{v_L^2}{2g}\right]-\frac{\gamma_G}{\gamma_L}\frac{d}{dx}\left[\frac{v_G^2}{2g}\right]+\frac{dh}{dx}-\sin\alpha+\frac{dh_m^{(L)}}{dx}-\frac{\gamma_G}{\gamma_L}\frac{dh_m^{(G)}}{dx}=0. \qquad (5)$$

It should be noted that this equation distinguishes itself from the equation of open channel flow through the influence that the gas flow has on the liquid flow. The influence occurs through the tangential as well as the normal stresses acting on the interphase.

THE BASIC ASSUMPTIONS

The basic equation can only be solved if certain assumptions are introduced.

(1) The flow of both the liquid and the gas phase is assumed to take place as incompressible fluids.

(2) The volume flow of liquid and gas is given as Q_L and Q_G, respectively. Using the notation F_L and F_G for those parts of the pipe cross section occupied by liquid and gas, respectively, one may give the mean velocities of the two phases as

$$v_L=\frac{Q_L}{F_L},\quad v_G=\frac{Q_G}{F_G}. \qquad (6)$$

(3) The head losses $h_m^{(L)}$ and $h_m^{(G)}$ will be introduced as if one had ordinary pipe flow

$$dh_m^{(L)}=f_L\frac{v_L^2\,dx}{4R_Lg}\pm f_i\frac{(v_L-v_G)^2\,dx}{4s_ig}, \qquad (7)$$

$$dh_m^{(G)}=f_G\frac{v_G^2\,dx}{4R_Gg}\mp f_i\frac{(v_L-v_G)^2\,dx}{4s_ig}. \qquad (8)$$

The first terms in equations (7) and (8) originate from the periphery of the liquid and the gas, respectively. The second terms originate from the conditions at the interphase between liquid and gas. If the gas moves faster than the liquid, the gas is 'pulling' the liquid along and the head loss is then positive for the gas but negative

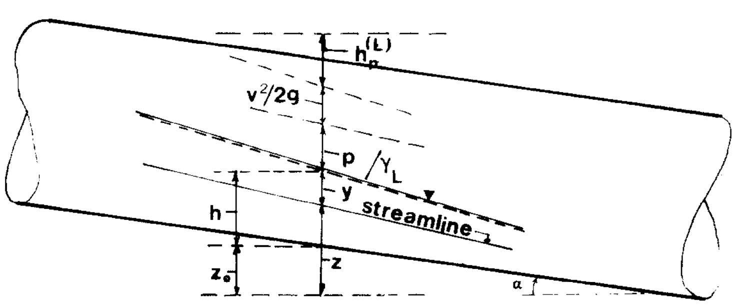

FIG. 2. The accelerated flow in a pipe.

for the liquid. If the gas moves slower, the opposite will be the case. Thus the signs of the last terms will have to be adjusted accordingly. (The upper sign if the gas moves slower.)

It has been assumed that the head losses are proportional to the square of the velocity as in rough pipes at high enough Re numbers, and that the head loss caused by the interphase between liquid and gas is proportional to the square of the velocity difference between the two phases. This implies that

$$f_L = f_G = f \neq f_i.$$

These assumptions may now be introduced in the basic equation, equation (5), which then may be written as

$$\frac{Q_L^2}{2g}\frac{d}{dx}\left(\frac{1}{F_L^2}\right) - \frac{\gamma_G}{\gamma_L}\frac{Q_G^2}{2g}\frac{d}{dx}\left(\frac{1}{F_G^2}\right) + \frac{dh}{dx}$$

$$= \sin\alpha - f\frac{Q_L^2}{4F_L^2 R_L g} + f\frac{\gamma_G}{\gamma_L}\frac{Q_G^2}{4F_G^2 R_G g}$$

$$\mp \frac{f_i}{4s_i g}\left(\frac{Q_L}{F_L} - \frac{Q_G}{F_G}\right)^2 \cdot \left(1 + \frac{\gamma_G}{\gamma_L}\right). \quad (9)$$

One may at this point introduce the circular pipe and correlate all quantities at one cross section with the depth h at the centreline of the liquid phase. From Fig. 3 one obtains the following relations

cross-sectional areas:

$$\left.\begin{aligned} F_L^+ &= F_L/r^2 = \phi - \sin\phi\cos\phi, \\ F_G^+ &= F_G/r^2 = \pi - F_L^+; \end{aligned}\right\} \quad (10)$$

wetted peripheries:

$$\left.\begin{aligned} P_L^+ &= P_L/r = 2\phi, \\ P_G^+ &= P_G/r = 2(\pi - \phi); \end{aligned}\right\} \quad (11)$$

hydraulic radii:

$$\left.\begin{aligned} R_L^+ &= F_L/P_L r = (\phi - \sin\phi\cos\phi)/2\phi, \\ R_G^+ &= F_G/P_G r = (\pi - F_L^+)/2(\pi - \phi); \end{aligned}\right\} \quad (12)$$

interphase:

$$s_i^+ = s_i/r = 2\sin\phi; \quad (13)$$

depth:

$$\cos\phi = 1 - h/r \Rightarrow \phi = \arccos(1 - h/r). \quad (14)$$

Since now the non-dimensional areas F_L^+ and F_G^+ are determined through the parameter ϕ, one may differentiate accordingly

$$\frac{d}{dx}\left(\frac{1}{F_L^2}\right) = -\frac{2}{F_L^3}\frac{dF_L}{d\phi}\cdot\frac{d\phi}{dh}\cdot\frac{dh}{dx},$$

where

$$\frac{dF_L}{d\phi} = r^2 2\sin^2\phi,$$

$$\frac{d\phi}{dh} = 1/r\sin\phi,$$

and consequently

$$\left.\begin{aligned} \frac{d}{dx}\left(\frac{1}{F_L^2}\right) &= -\frac{4}{r^5(F_L^+)^3}\sin\phi\frac{dh}{dx}, \\ \frac{d}{dx}\left(\frac{1}{F_G^2}\right) &= \frac{4}{r^5(F_G^+)^3}\sin\phi\frac{dh}{dx}. \end{aligned}\right\} \quad (15)$$

It has been shown how the cross-sectional quantities have been non-dimensionalized, the dimensionless quantities being noted by the superscript (+). It is now possible to non-dimensionalize also the depth h and the distance x along the pipe by means of the radius of the pipe

$$h^+ = h/r, \quad x^+ = x/r. \quad (16)$$

Then the basic equation, equation (9), may after some routine algebra be written in the form

$$\frac{dh^+}{dx^+}\left\{1 - (h_{crit}^+)^3\left[1 + P_2 P_1^2\left(\frac{F_L^+}{F_G^+}\right)^3\right]\frac{\sin\phi}{(F_L^+)^3}\right\}$$

$$= \sin\alpha\left\{1 - (h_0^+)^3\left[\frac{1}{(F_L^+)^2 R_L^+} - \frac{P_2 P_1^2}{(F_G^+)^2 R_G^+}\right.\right.$$

$$\left.\left.\pm \frac{P_4}{s_i^+}\left(\frac{1}{F_L^+} - \frac{P_1}{F_G^+}\right)^2(1 + P_2)\right]\right\}, \quad (17)$$

where the following dimensionless parameters have been introduced

$$\left.\begin{aligned} (h_{crit}^+)^3 &= \frac{2Q_L^2}{gr^5}, \\ (h_0^+)^3 &= \frac{fQ_L^2}{g4r^5\sin\alpha}, \\ P_1 &= Q_G/Q_L, \\ P_2 &= \gamma_G/\gamma_L, \\ P_4 &= f_i/f. \end{aligned}\right\} \quad (18)$$

Equation (17) represents the basic equation of the flow from which certain characteristics of the flow may easily be deduced even without solving it.

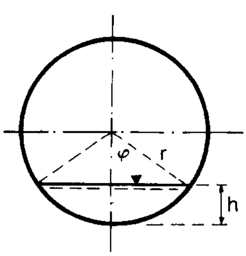

FIG. 3. Cross section of the pipe.

DISCUSSION OF THE BASIC EQUATION. THE NORMAL DEPTH

The 'normal depth' h_n of the liquid is defined as that value of h for which $dh^+/dx^+ = 0$. Once h has attained this value h_n it will retain it. This implies that the RHS of equation (17) must be zero, and this becomes the condition which determines h_n. It is observed that h_n thus will depend on the four parameters h_0^+, P_1, P_2 and P_4. Since all cross-sectional quantities are expressed through ϕ, the value of $\phi = \phi_n$ corresponding to h_n will be determined as the root of the following equation

$$1 = (h_0^+)^3\left[\frac{1}{(F_L^+)^2 R_L^+} - \frac{P_1^2 P_2}{(F_G^+)^2 R_G^+} \pm \frac{P_4}{s_i^+}\left(\frac{1}{F_L^+} - \frac{P_1}{F_G^+}\right)^2 (1+P_2)\right]. \quad (19)$$

It must be realized that the upper and lower signs of the last term in the square bracket must be chosen according to the rules mentioned earlier, i.e.

$$\left.\begin{matrix}\text{upper}\\ \text{lower}\end{matrix}\right\} \text{sign when } \frac{v_L}{v_G} = \frac{1}{P_1}\cdot\frac{F_G^+}{F_L^+}\begin{cases}>1\\ <1\end{cases}. \quad (20)$$

The solution to equation (19) may now be illuminated by a few worked examples exhibited in Figs. 4(a)–(c). In these figures the normal depth $h_n^+ = h_n/r$ is given as a function of h_0^+ with P_1 as a parameter. In each figure the two other parameters (P_2 and P_4) have been kept constant.

The parameter P_2 will under most conditions be rather small. For air–water flow at atmospheric pressure and 20°C one will have

$$P_2 = 0.00121.$$

Figures 4(a)–(c) have been computed with this value. It may be mentioned that as long as P_2 is in the range $0.01 < P_2 < 0.001$ an influence of a variation in P_2 can hardly be traced in the figures. Figures 4(a)–(c) are computed for different values of P_4 in order to give an impression of the influence of the physical conditions at the interphase expressed through this parameter. (It

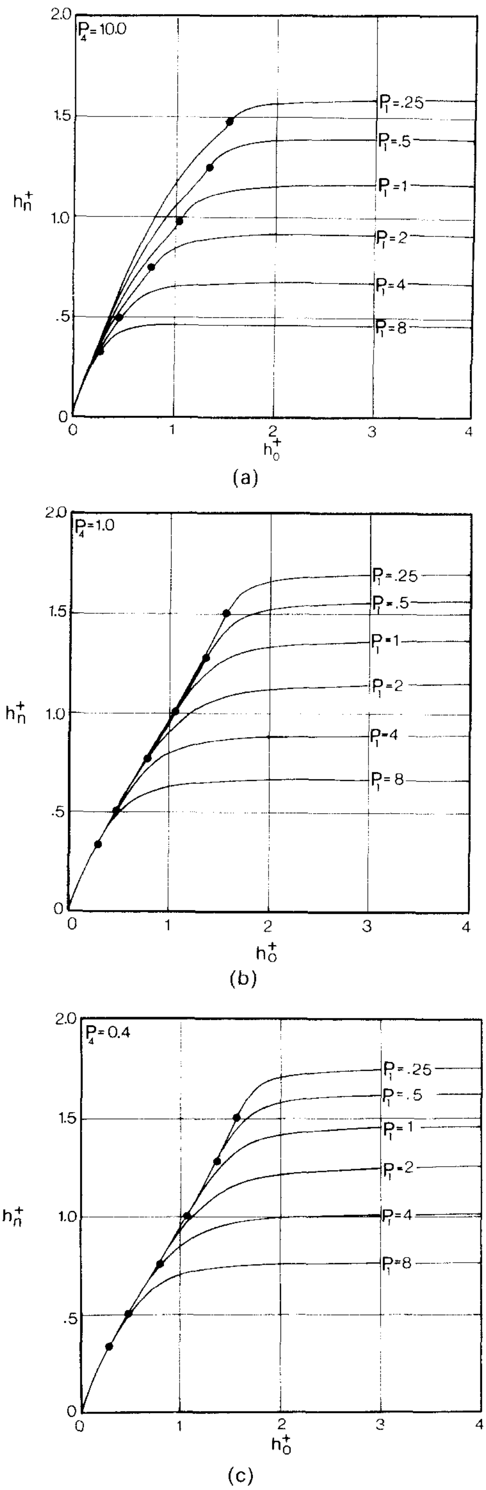

FIG. 4(a)–(c). Normal depth h_n^+ vs h_0^+ for various values of P_1 ($P_2 = 0.00121$).

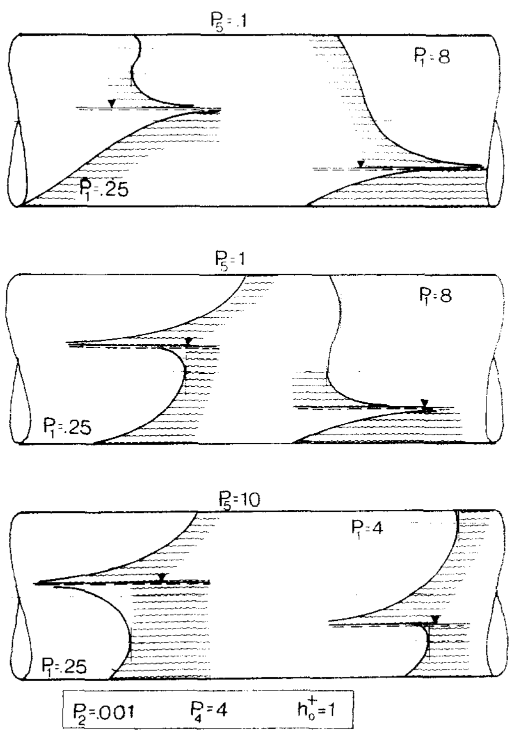

FIG. 5. Typical behaviour of the interphase for given values of the parameters.

may be mentioned that this parameter may be expected to be in the range $0.01 < P_4 < 10$ depending on the flow conditions.)

The parameter P_1 representing the ratio between the volumetric flow of gas and of liquid is seen to have a large effect on the asymptotic value of the normal depth h_n^+ for large values of h_0^+. This latter parameter increases with the volumetric flow of liquid, i.e. when the ratio P_1 is constant, h_0^+ may be taken as a measure of the total flow. Thus, in view of the asymptotic behaviour of the curves, the normal depth h_n^+ for large values of h_0^+ will be determined by P_1 and P_4 only.

It ought to be mentioned that in Figs. 4(a)–(c) the value of P_2 may be changed by an order of magnitude with hardly any influence in the figure. Thus one may conclude that the normal depth h_n^+ seems to be mainly determined by the gas–liquid ratio, the condition at the interphase expressed through P_4 and the total flow expressed through h_0^+.

A final point to be made concerns the fact that under certain circumstances the gas will be 'pulling' the liquid and sometimes the reverse will be the case. In equation (23) the upper sign is to be chosen when the liquid pulls the gas. For a specified value of P_1 this will be the case to the left of the 'dot' on the curves in Figs. 4(a)–(c). To the right of the 'dot' the gas pulls the liquid.

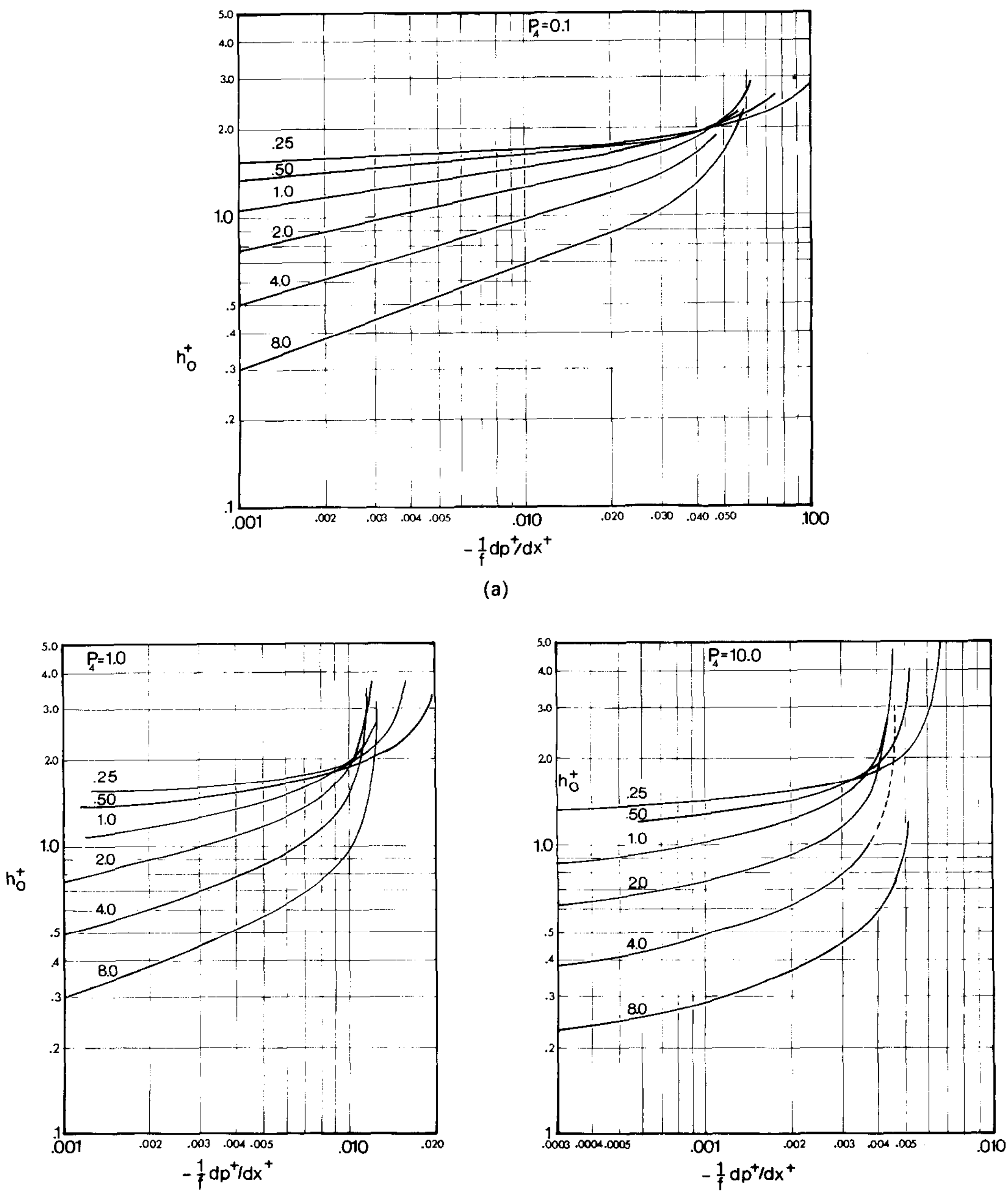

FIG. 6(a)–(c). Pressure gradient vs h_0^+ for various values of P_4 with P_1 as a parameter ($P_2 = 0.00121$).

Figures 4(a)–(c) showing how the normal depth h_n depends on the different parameters will have to be supplemented by information on the value of dh^+/dx^+ when h^+ deviates from the normal depth h_n^+. The situation in channel flow is such that a critical depth exists, and the behaviour of the surface when brought away from its equilibrium position (the normal depth) is different according to whether or not the normal depth is smaller or larger than the critical depth. A similar behaviour is to be expected also here. One may therefore conclude that a subcritical and supercritical behaviour of the interphase may be expected which will be caused by considerations other than those leading to 'stratified smooth' and 'stratified wavy' flow. The latter cases are caused mainly by the interaction at the interphase.

First it is observed that in elementary channel flow the difference between the two cases can be expressed through a 'critical slope' of the channel. In the present case it is observed that $h_{\rm crit}^+$ can be expressed as

$$(h_{\rm crit}^+)^3 = (h_0^+)^3 \cdot \frac{1}{P_5}, \tag{21}$$

where

$$P_5 = \frac{f}{8 \sin \alpha}. \tag{22}$$

Thus the parameter P_5 depends on the friction coefficient f and the slope of the channel $\sin \alpha$ only and will play the role of the 'criterion' which however in the present case only conditionally distinguishes between the two cases. Figures 5(a)–(c) exemplify the situation. The figures give sketches of the behaviour of the surface (interphase) once it is brought away from the normal depth. The forms are seen to depend on the parameters but certain general features may be brought out. In Fig. 5(a) the value of P_5 is so small (large slope) that whether $P_1 = 0.25$ or 8 makes no difference: the situation in both cases is that of supercritical flow. In Fig. 5(c) the value of P_5 is so large (small slope) that for both values of P_1 the behaviour of the surface (interphase) is that of subcritical flow. In Fig. 5(b), however, an intermediate value of P_5 has been chosen to show that changing the value of P_1 from 0.25 to 8 will change the flow from subcritical to supercritical. In accordance with the situation in channel flow the liquid may also here exhibit hydraulic jumps as indicated by the occurrence of vertical slopes of the interphase.

THE PRESSURE GRADIENT

One may now return to equation (2) and solve for the pressure gradient

$$\frac{1}{\gamma_L}\frac{dp_G}{dx} = \sin \alpha - \frac{dh}{dx} - \frac{d}{dx}\left[\frac{v_L^2}{2g}\right] - \frac{dh_m^{(L)}}{dx}. \tag{23}$$

Considering the case of uniform flow ($h = h_n$) where the variations of h and v_L with respect to x are zero, and utilizing equation (7) one obtains

$$\frac{1}{\gamma_L}\frac{dp_G}{dx} = \sin \alpha - f_L \frac{v_L^2}{4R_L g} \mp f_i \frac{(v_L - v_G)^2}{4s_i g}. \tag{24}$$

The pressure p_G is now non-dimensionalized in the following way

$$p_G^+ = p_G / \tfrac{1}{2}\rho_L v_L^2, \tag{25}$$

and consequently the pressure gradient may after some lengthy but straightforward algebra be reformulated as follows

$$\frac{dp_G^+}{dx^+} = \tfrac{1}{2} f (F_L^+)^2 \left\{ -\frac{P_1^2 P_2}{(F_G^+)^2 R_G^+} \pm \frac{P_4 P_2}{s_i^+}\left(\frac{1}{F_L^+} - \frac{P_1}{F_G^+}\right)^2 \right\}. \tag{26}$$

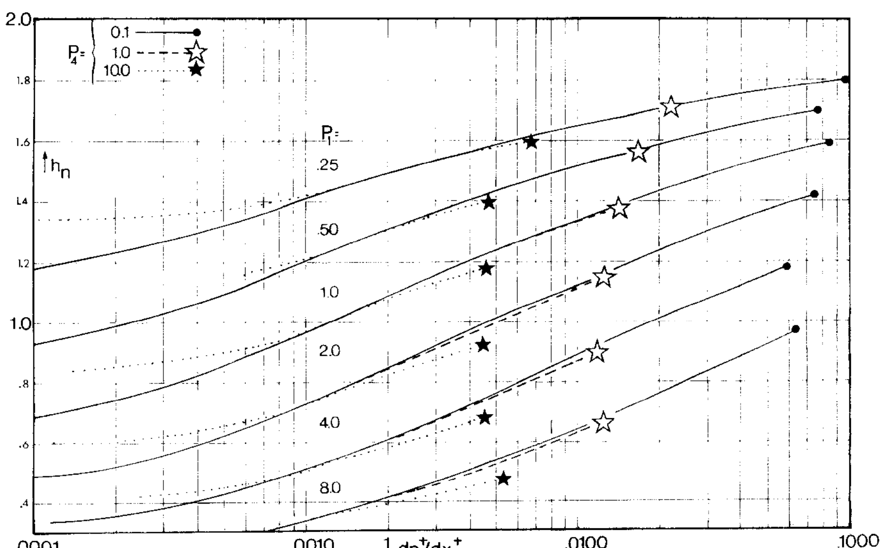

FIG. 7. Display of the pressure gradient as a function of the normal depth h_n for different values of the parameters P_1 and P_4.

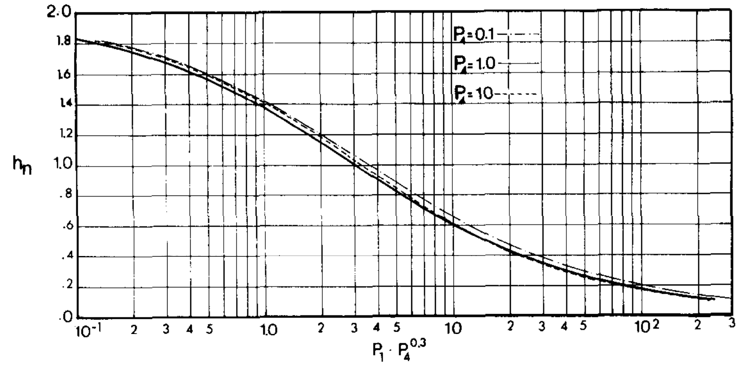

FIG. 8. The asymptotic value $h^+_{n\infty}$ of the normal depth h^+_n as a function of the parameters P_1 and P_4 with $P_2 = 0.00121$.

This expression for the pressure gradient is now exhibited in Figs. 6(a)–(c) where dp^+_G/dx^+ is shown in dependence of h^+_0 with P_1 as a parameter. The figures exhibit the influence of a change in P_4.

A slightly different perspective is perhaps obtained if the pressure gradient is plotted in relation to the normal depth. This is done for different values of the parameters P_1 and P_4 in Fig. 7. It is remarkable how close the curves for different values of P_4 merge over a large range. (The stars indicate the points at which h_n reaches its asymptotic value $h_{n\infty}$.) It is noted that only cases where the gas 'pulls' the liquid are considered.

A final remark may be made. It is observed from Figs. 4(a)–(c) that h^+_n reaches its asymptotic value $h_{n\infty}$ for rather small values of h^+_0. If one assumes $h^+_0 \to \infty$ one will from equation (23) find that

$$\frac{1}{(F^+_L)^2 R^+_L} - \frac{P_1^2 P_2}{(F^+_G)^2 R^+_G} - \frac{P_4}{s^+_i}\left(\frac{1}{F^+_L} - \frac{P_1}{F^+_G}\right)^2 (1+P_2) = 0. \tag{27}$$

In this expression $F^+_L, R^+_L, F^+_G, R^+_G$ and s^+_i have the value obtained by introducing $h^+_{n\infty}$ for h^+ into equations (10), (12) and (13). The equation can thus be conceived of as an expression of the dependence of $h^+_{n\infty}$ on the three parameters P_1, P_2 and P_4. This expression is exhibited in Fig. 8 where the argument $P_1 \cdot P_4^{0.3}$ has been introduced to obtain a display that changes only slightly within the variation of the parameters P_1 and P_4. This may be utilized to determine experimentally the value of P_4. The normal depth $h^+_{n\infty}$ can easily be measured, and since the value of P_1 is known P_4 can be deduced. Such experiments are already being conducted at the laboratories of the Institutt for Mekanikk, NTH, University of Tronheim.

FINAL REMARKS

It should be observed that the present approach to the problem of stratified two-phase flow is strictly 1-D and that consequently questions of detailed behaviour of the interphase, the velocity distribution in the two phases, and similar questions cannot be answered. The forces acting from one phase to the other as well as on the walls are gross effects and functions of the mean velocities of the two phases. Such physical parameters as f and f_i must therefore be determined experimentally as has already been pointed out.

It should also be noted that the accelerated steady flow which has been used as the basis for the present considerations may be difficult to maintain experimentally. However, situations occurring when the slope of almost horizontal pipes are suddenly changed will represent cases where the present approach becomes important for steady flow conditions. It is also stressed that the approach accounts for different pipe sizes through the parameter h^+_0. The question of scaling to different pipe sizes should therefore be no problem as long as applied within the regions set by the basic assumptions underlying the approach.

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ECOULEMENT DIPHASIQUE STRATIFIE DANS LES TUBES CIRCULAIRES

Résumé—On étudie l'écoulement stratifié dans un tube circulaire de liquide et de gaz qui ne réagissent pas entre eux. Quand un débit massique se produit à travers les phases, l'influence de l'entrainement des gouttes de liquide dans le gaz ou le dépôt de quelques unes sur les parois ou sur l'interface est prise en compte par un paramètre qui exprime l'effet intégral de ce phénomène. L'inclusion de cette configuration de l'écoulement a un effet dramatique sur la profondeur normale de la phase liquide sous des conditions données. On donne des équations analytiques pour la perte de charge et les conditions qui déterminent la forme de l'interface incluant des "sauts hydrauliques". Une approche particulière concerne le cas de la variation de pression dans la phase liquide due à la pesanteur. Les résultats peuvent être utilisés comme base d'une évaluation expérimentale de la tension de cisaillement à l'interphase.

ZWEIPHASEN-SCHICHTEN-STRÖMUNG IN KREISROHREN

Zusammenfassung—Die Schichten-Strömung von Flüssigkeit und Gas in einem Kreisrohr wird unter der Bedingung untersucht, dass die beiden Komponenten nicht miteinander reagieren. Wenn ein Massenstrom über die Grenzfläche auftritt, wird der Einfluss des Mitreissens von Flüssigkeits-Tröpfchen in das Gas, oder ihre Wiederablagerung an der Wand oder der Grenzfläche durch einen konzentrierten Parameter berücksichtigt, der den integralen Effekt dieses Vorganges wiedergibt. Es wird gezeigt, dass die Berücksichtigung dieser Eigenschaft der Strömung unter gegebenen Bedingungen einen ausgeprägten Einfluss auf die Normal-Tiefe der flüssigen Phase hat. Für den Druckabfall und die die Gestalt der Grenzfläche—einschliesslich möglicher 'Wassersprünge'—bestimmenden Bedingungen werden explizite analytische Beziehungen angegeben. Einen Eigenheit des Ansatzes ist die Art, in der die Änderung des Druckes in der flüssigen Phase aufgrund der Schwere berücksichtigt wird. Zu erwähnen ist, dass die Ergebnisse der Arbeit als Grundlage für die experimentelle Auswertung der integrierten Schubspannung an der Grenzfläche verwendet werden können.

СТРАТИФИЦИРОВАННОЕ ДВУХФАЗНОЕ ТЕЧЕНИЕ В КРУГЛЫХ ТРУБАХ

Аннотация—Рассматривается стратифицированное течение жидкости и газа в круглой трубе при условии, что оба компонента не вступают между собой в химическую реакцию. В случае, если через границу раздела происходит перенос массы, влияние попадания капель жидкости в газ или оседания их на стенках трубы или на границе раздела учитывается с помощью эффективного параметра. Показано, что учет этого явления оказывает существенное влияние на толщину слоя жидкой фазы в таких условиях. Представлены аналитические уравнения в явном виде для определения перепадов давления и условий, влияющих на форму границы раздела, включая возможные "гидравлические всплески". Особенностью данного подхода является способ, каким определялось изменение давления в жидкой фазе из-за влияния силы тяжести. Результаты работы могут использоваться для экспериментальной оценки суммарного напряжения сдвига на границе раздела.